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BACKSCATTERED SPECTRA FROM ROTATING AND VIBRATING SHORT WIRES AND THEIR RELATION TO THE IDENTIFICATION PROBLEM

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In this report Rayleigh scattering for a short wire is examined for two simple motions, rotation and vibration. Rotation of the wire causes an amplitude modulation and vibration causes a phase modulation. Each of these modulations influences the backscattered spectra so that the characteristic frequency of the periodic motion appears. For the rotating wire, it is twice the rotation frequency. From the spectra it is possible to recognize the type of motion responsible. The study therefore serves as a first step in the development of

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models for complicated motions of actual objects such as tanks and trucks, thereby leading to possible vehicle signature classification schemes. The appendices include: a brief discussion of the formalism of polarization scattering matrices; a short examination of the relation between the scattered spectra due to periodic motions and the conventional Doppler effect; and an examination of the analogy between the rotating and vibrating wires and molecules as conventional Raman scatterers, including a brief section on the energetics of the scattering process.

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Backscattered Spectra From Rotating and Vibrating Short Wires and Their Relation to the Identification Problem

I. INTRODUCTION

Radar reflections may be related to the specific nature of a target–its structure, configuration, and internal motion. If targets such as trucks, tanks, and helicopters are illuminated with radar, the reflections contain spectral components which are distinct from the Doppler shifts arising from simple translatory motion. For example it is well-known that a rotating helicopter blade modulates the radar reflections from the helicopter with a frequency related to the rotation rate of the blade. Vibrating membranes, because of their time varying position, also cause modulation in the phase of the target return which results in frequency shifts. The use of these spectral changes in the reflections to classify radar signatures of various targets having internal vibrations and rotations may have possible applications in vehicle classification.

Many military vehicles have periodic motions, internal and external, with definite frequencies. These motions affect the radar reflections and may be used in vehicle identification. Therefore, it would seem that an understanding of these various phenomena must include an understanding of the contributions of the vehicular rotations and vibrations. In this paper we examine a short wire undergoing two exceedingly simple motions, rotation and vibration. In these motions the wire itself is taken to be rigid. The shortness of the wire–that is, short

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compared with the wavelength of the radar—means that we can treat the wire as a Rayleigh scatterer. We first determine the scattering matrices for these motions, and thus obtain the scattered fields for different polarizations of the incident field. Applying Fourier techniques to the scattered fields gives the frequency components.

The rotation and vibration spectra are quite different, indicating possibilities for signature classifications. The differences also suggest possible directions, both experimental and analytical, for future studies. These would include combining rotation and vibration, analysis of more complicated structures such as plates, propellers, and belts, and studies outside the Rayleigh region where the object dimensions are no longer small compared with the wavelength. The ultimate goal is to be able to synthesize the radar returns from real vehicles, from a set of relatively simple models.

There are several appendices. The first is mathematical and describes the polarization of radiation in terms of a cartesian basis or an equivalent circular basis. With these bases, the formalism becomes quite simple and makes it easy to set up the polarization scattering matrix. The derivation of the matrix and its use are discussed in the same appendix. The three additional appendices deal with the physical interpretation of the scattering from the wire. The rotating or vibrating wire may be looked on as a microwave model for Raman molecular spectroscopy. Although the correspondence between the two is not perfect—since Raman spectroscopy is a quantum effect—a discussion of one in terms of the other is illuminating. A second point is the relation of scattering from the rotating or vibrating wire to the normal Doppler effect. It is true that the rotation and vibration lead to definite frequency shifts, but they are not the same as those found with purely translating objects. For clarity in discussion the difference must be made evident. The final appendix contains a brief discussion of the energetic and angular momentum transfers in the scattering process.

2. SCATTERING FROM A ROTATING WIRE

As stated in the introduction, we restrict ourselves in this report to two very simple motions. In this section we consider a rotating wire which is short compared with the radiating wavelength. Physically this means that the wire acts as a point dipole, and that only the lowest electric mode as defined by Harrington and Mautz[1] is excited. Equivalently we can say that the wire acts as a Rayleigh scatterer.

1. Harrington, R.F., and Mautz, J.R. (1971) IEEE Trans. Antennas Propag. AP-19:622.

Schoendorf[2] has treated the problem for a rotating short wire illuminated by a circularly polarized wave. The polarization state of the incident field will prove to be important, whether it be linearly, circularly, or elliptically polarized. Consider a wire rotating in the xy-plane at constant angular velocity $\dot{\theta}$ as shown in Figure 1. The incident radiation is monochromatic with frequency ν_0. The direction of propagation is along the z-axis, that is, the wave vector $\underset{\sim}{k}$ is normal to the plane of rotation of the wire, and k_x and k_y are both zero. For this geometry, the electric field of the incident wave has no z-component. Maxwell's equations impose the requirement of continuity of the tangential components of the electric field at a boundary. Because of the geometry, the current induced by the incident field must flow along the wire's axis. Therefore the scattered radiation must be linearly polarized, since the field in the far zone has one component only. This is independent of the polarization of the incident field. Therefore the reflected or scattered field in the far zone always has its electric vector parallel to the wire at the instant of reflection. Naturally this implies that $\dot{\theta}$ is much smaller than ν_0, which holds for all realizable cases. However, since the wire rotates, the electric vector reflected at each instant will be linearly polarized in a direction which depends on the temporal orientation of the wire at that instant. It is this changing orientation which introduces the frequency $\dot{\theta}$ into the spectrum of the reflected field.

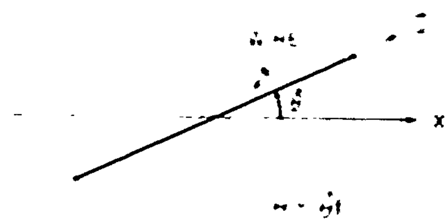

Figure 1. Wire Rotating in the xy-plane With Angular Velocity $\dot{\theta}$

To calculate the effects of rotation, we first write the incident and scattered waves in terms of the cartesian basis $(\hat{x},\hat{y})$ described in Appendix A,

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2. Schoendorf, W.H. (1972) Frequency Spectrum and Backscattered Return From a Rotating Short Wire, PA-267, Lincoln Laboratory, M.I.T., Cambridge, Massachusetts.

$$\underset{\sim}{E}^i = \left[E_x^i \hat{x} + E_y^i \hat{y}\right] \exp\left[i(kz - \omega_o t)\right],$$

and

$$\underset{\sim}{E}^s = \left[E_x^s \hat{x} + E_y^s \hat{y}\right] \exp\left[i(kz - \omega_o t)\right]. \tag{1}$$

The unit vector along the wire is $\hat{u}$, and θ is the angle between the wire and the x-axis. The wire rotates with angular velocity $\dot{\theta}$; therefore θ equals $\dot{\theta}$ t. Owing to the requirement of continuity of the tangential components of $\underset{\sim}{E}$ at a surface, we can write $\underset{\sim}{E}^s$ in terms of $\underset{\sim}{E}^i$ as

$$\begin{aligned}\underset{\sim}{E}^s &= C\,(\underset{\sim}{E}^i \cdot \hat{u})\,\hat{u} \\ &= C\left[(\hat{x}\cdot\hat{u})\,E_x^i + (\hat{y}\cdot\hat{u})\,E_y^i\right]\hat{u},\end{aligned} \tag{2}$$

a field linearly polarized in the direction of the wire. In Eq. (2), C is a proportionality constant. By taking the x and y components of $\underset{\sim}{E}^s$, we find

$$E_x^s = C\left[(\hat{x}\cdot\hat{u})^2 E_x^i + (\hat{x}\cdot\hat{u})(\hat{y}\cdot\hat{u})\,E_y^i\right],$$

and

$$E_y^s = C\left[(\hat{x}\cdot\hat{u})(\hat{y}\cdot\hat{u})E_x^i + (\hat{y}\cdot\hat{u})^2 E_y^i\right]. \tag{3a}$$

But, Eq. (3a) may be written in the matrix form of Eq. (A2) in Appendix A

$$\begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} = CR \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} \tag{3b}$$

where [R] is the scattering matrix. Since

$$(\hat{x}\cdot\hat{u}) = \cos\theta = \cos(\dot{\theta}\,t) \text{ etc.}, \tag{4}$$

Eqs. (3a) and (3b) enable us to determine the matrix elements r_{ij} of the scattering matrix [R],

$$r_{11} = (\hat{x}\cdot\hat{u})^2 = \cos^2(\dot{\theta}t)$$

$$r_{22} = (\hat{y}\cdot\hat{u})^2 = \sin^2(\dot{\theta}t)$$

$$r_{12} = r_{21} = (\hat{x}\cdot\hat{u})(\hat{y}\cdot\hat{u}) = \sin(\dot{\theta}t)\cos(\dot{\theta}t) \qquad (5)$$

$$= 1/2\sin(2\dot{\theta}t).$$

Equations (3a) and (3b) relate the scattered field to the incident field, provided each is expressed in terms of the cartesian basis $(\hat{x}, \hat{y})$. If the incident field is expressed in terms of the circular polarization basis $(\hat{L}, \hat{R})$ and we wish to determine the reflected field in terms of the same basis, we apply Eq. (A15) of Appendix A.

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = C Q^{-1} R Q \begin{bmatrix} E_L^i \\ E_R^i \end{bmatrix}. \qquad (6)$$

The circular scattering matrix Q^{-1} R Q is easily calculated to be

$$Q^{-1} R Q = 1/2 \begin{bmatrix} (r_{11} + r_{22}) & (r_{11} - r_{22} + 2 i r_{12}) \\ (r_{11} - r_{22} - 2 i r_{12}) & (r_{11} + r_{22}) \end{bmatrix}. \qquad (7)$$

With the help of Eqs. (5) and (7) we can determine the backscattered field for the above geometry, when the incident field is known. The main objective, however, is the determination of the spectra of the scattered fields. To obtain the spectra or frequency dependence of these fields, we apply Fourier techniques. Since delta functions allow us to use Fourier transforms for periodic functions as well as aperiodic ones, the Fourier series becomes a special case of the transform. The convolution theorem states that the Fourier transform of a product of two functions is proportional to the convolution of their individual transforms. Since the temporal functions we deal with are simple trigonometric functions, their transforms are combinations of appropriately weighted delta functions. The trigonometric functions and their transforms are shown in Figure 2. The convolutions are easily calculated graphically and are shown in Figure 3. By adding the various convolutions we obtain the desired spectra.

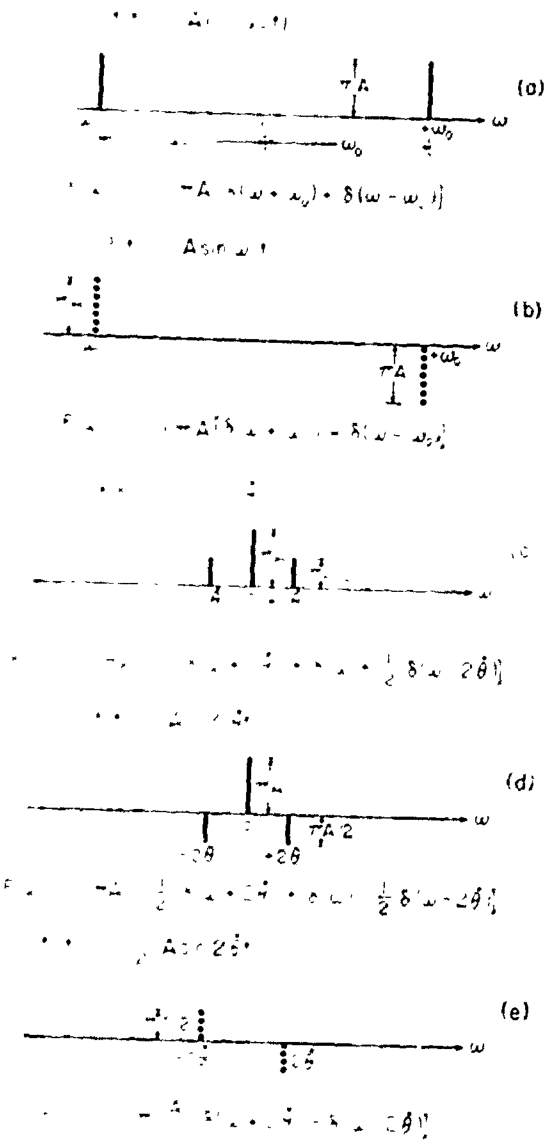


Figure 2. Fourier Transforms $F(\omega)$ of Various Trigonometric Functions f(t). The dotted lines indicate imaginary quantities

We now consider three examples with the rotating wire.

Example 1: The wire rotates in the xy-plane with uniform velocity $\dot{\theta}$. The incident field is left circularly polarized (LCP), of unit magnitude, and

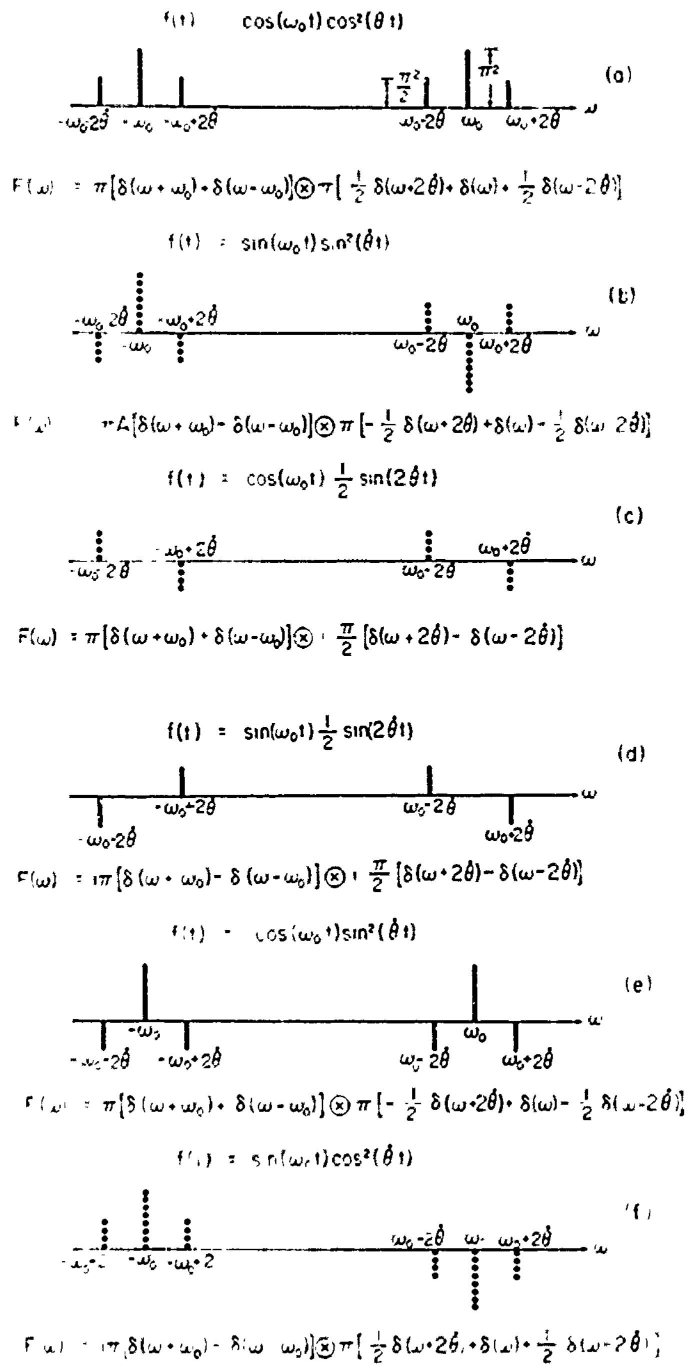


Figure 3. Fourier Transforms F(ω) of the Temporal Functions Relevant to the Rotating Wire. The dotted lines indicate imaginary quantities

propagates in the z-direction,

$$\underset{\sim}{E}^{i} = \hat{L} \exp[i(kz - \omega_o t)]. \tag{8}$$

In terms of Eq. (A9) of Appendix A, E_L equals unity and E_R equals zero. The scattered field

$$\underset{\sim}{E}^{s} = \left[E_L^s \hat{L} + E_R^s \hat{R}\right] \exp[-i(kz + \omega_o t)] \tag{9}$$

is computed from Eqs. (6) and (7), and the amplitudes are found to be

$$E_L^s = (C/2)(r_{11} + r_{12}), \text{ and}$$

$$E_R^s = (C/2)(r_{11} - r_{22} - 2ir_{12}), \tag{10}$$

where the r_{ij} are given by Eq. (5). It is clear, therefore, that, although the incident field is in a pure LCP state, the reflected field is a mixture of LCP and RCP (right circularly polarized) states. Since, from Eq. (5)

$$|r_{11} - r_{22} - 2ir_{12}| = |r_{11} + r_{22}|,$$

$$|E_L^s| \big/ |E_R^s| = 1. \tag{11a}$$

The left and right components are therefore equal, which we know must be true because of the linear polarization of the scattered field.

We can also obtain the equality of the two components from the criterion of Rayleigh scattering. Since the scattered power is proportional to the fourth power of frequency, we can write

$$|E_L|^2 \big/ |E_R|^2 = \left[(\omega_o + 2\dot{\theta}) / \omega_o\right]^4. \tag{11b}$$

Since $\dot{\theta}$ is much smaller than ω_o, the ratio is approximately unity.

The time dependent components of the reflected field are proportional to $(r_{11} + r_{22}) \exp(-i\omega_o t)$ and $(r_{11} - r_{22} - 2ir_{12}) \exp(-i\omega_o t)$.

The real part of these expressions is

$$\mathrm{Re}[E^s(t)] = C[2\cos^2(\dot{\theta}t)\cos(\omega_o t)$$

$$- \sin(2\dot{\theta}t)\sin(\omega_o t)]. \tag{12}$$

The Fourier transform of Eq. (11), computed easily from Figures (3a) and (3d), indicates that the spectrum consists of two lines of equal strength at the two frequencies ω_o and $(\omega_o + 2\dot{\theta})$. Each line is of different polarization, the shifted one being of the polarization admitted by the transmitting antenna. The contributions at $(\omega_o - 2\dot{\theta})$ cancel.

If the incident field were RCP of unit magnitude and propagating in the z-direction (so that E_R were unity and E_L zero), we would again obtain two spectral lines but at $(\omega_o - 2\dot{\theta})$ and ω_o.

Example 2: The wire rotates in the xy-plane with uniform velocity $\dot{\theta}$, just as in Example 1. The incident field is still of unit magnitude but is now linearly polarized.

$$\underset{\sim}{E}^i = \hat{x}\exp[i(kz - \omega_o t)]. \tag{13}$$

Therefore E_x is unity and E_y zero. To express the scattered field in terms of $\hat{L}$ and $\hat{R}$, we apply Eqs. (A12) and (A14a)

$$\begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} = C R \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix}$$

and (14)

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = Q^{-1} \begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} .$$

Therefore,

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = C Q^{-1} R \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = C Q^{-1} R \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{15}$$

which yields

$$E^s_L = (C/\sqrt{2})(r_{11} + ir_{12}), \text{ and}$$

$$E^s_R = (C/\sqrt{2})(r_{11} - ir_{12})$$

where the r_{ij} are given by Eq. (5) as before. Again the magnitudes of the two components are equal, and the reflected field is linearly polarized.

The time-dependent components of the reflected field are proportional to $(r_{11} \pm ir_{12})\exp(-i\omega_o t)$. The real part of these expressions is

$$\mathrm{Re}\,[E^s(t)] = C\,2\cos^2(\dot{\theta}\,t)\cos(\omega_o t). \tag{16}$$

The Fourier transform of Eq. (16), computed from Figure (3a), indicates that the spectrum consists of three lines at ω_o and $(\omega_o \pm 2\dot{\theta})$. The side lines are each half the magnitude of the center frequency line at ω_o.

As might be expected, this is exactly the result we would obtain by adding the spectra of a unit LCP and a unit RCP field as computed in Example 1. For a linearly polarized field is decomposable into equal L and R components.

Example 3: The wire rotates with an angular velocity $\dot{\theta}$ in a plane perpendicular to axis $\hat{p}$ shown in Figure 4. The axis makes an arbitrary angle ψ with the direction of propagation which coincides with the z-axis. The unit rotation axis $\hat{p}$ has components $(0, \sin\psi, \cos\psi)$. The axis of the wire is always normal to $\hat{p}$, and at time t_o such that $\dot{\theta}\,t_o$ equals $\pi/2$ it lies in the yz-plane. Therefore, $\hat{u}_o$ which equals $\hat{u}(t_o)$ has components $(0, +\sin\psi, \cos\psi)$. As the wire rotates it acquires an x-component, and the general time dependence of the wire's direction is

$$\begin{aligned}\hat{u}(t) &= \hat{x}\cos(\dot{\theta}\,t) + \hat{u}_o\sin(\dot{\theta}\,t)\\ &= \hat{x}\cos(\dot{\theta}\,t) + \hat{y}\sin\psi\,\sin(\dot{\theta}\,t) - \hat{z}\cos\psi\,\sin(\dot{\theta}\,t).\end{aligned} \tag{17}$$

Substituting this value into Eq. (5), we obtain for the elements of the scattering matrix

$$r_{11} = (\hat{x} \cdot \hat{u})^2 = \cos^2(\dot{\theta} t),$$

$$r_{12} = (\hat{x} \cdot \hat{u})(\hat{y} \cdot \hat{u}) = \sin\psi \sin(\dot{\theta} t)\cos(\dot{\theta} t)$$

$$= 1/2 \sin\psi \sin(2\dot{\theta} t), \text{ and}$$

$$r_{22} = (\hat{y} \cdot \hat{u})^2 = \sin^2\psi \sin^2(\dot{\theta} t). \tag{18}$$

If we compare Eq. (18) with Eq. (5), we see that the time dependency of the scattering matrix has remained unchanged. Therefore, there will be no chang in the frequency components of the spectra as compared with those obtained in Examples 1 and 2.

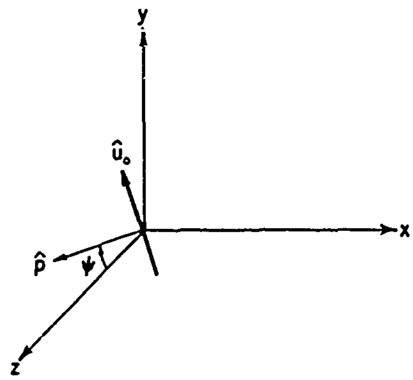


Figure 4. Wire Rotating About p-direction Which Makes an Angle ψ With the Direction of Propagation. At instant shown the wire direction $\hat{u}_o$ is in the yz-plane

3. SCATTERING FROM A VIBRATING WIRE

The second motion considered in this report is vibration. As before, we shall examine a short wire which acts as a point dipole. Let the axis of the wire be oriented parallel to the x-axis of Figure 1, so that the wire vibrates in the z-direction with an oscillation frequency $\dot{\theta}$. The position of the wire as a function of time is, therefore,

$$z = z_o \sin(\dot{\theta} t). \tag{19}$$

Again, as in Section 2, the incident radiation is monochromatic with frequency ω_o ($\omega_o >> \dot{\theta}$) and propagates in the z-direction. Consequently, the motion of the wire is parallel to the direction of propagation, always advancing or retreating with respect to the source.

To compute the elements of $[R]$, we remember that the axis of the wire $\hat{u}$ is parallel to x. The elements of the matrix are

$$r_{11} = (\hat{x} \cdot \hat{u}) = 1,$$

$$r_{22} = (\hat{y} \cdot \hat{u}) = 0, \text{ and}$$

$$r_{12} = (\hat{x} \cdot \hat{u})\,(\hat{y} \cdot \hat{u}) = 0, \tag{20}$$

since $(\hat{y} \cdot \hat{u})$ equals zero at all times for this geometry. The matrix $Q^{-1} R Q$ is therefore

$$Q^{-1} R Q = 1/2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

and

$$Q^{-1} R = \left(1/\sqrt{2}\right) \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \tag{21}$$

both of which are time independent. We now apply these matrices to Examples 1 and 2 of Section 2.

<u>Example 1:</u> The incident field is LCP, of unit magnitude, and propagates in the z-direction.

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = C\,Q^{-1} R Q \begin{bmatrix} E_L^i \\ E_R^i \end{bmatrix}, \text{ and}$$

$$= (C/2) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (C/2) \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{22}$$

The scattered field is linearly polarized, since E_L and E_R are equal.

<u>Example 2:</u> The incident field is linearly polarized (with E_y equal to zero), of unit magnitude, and propagates in the z-direction.

$$
\begin{bmatrix} E^s_L \\ E^s_R \end{bmatrix} = C Q^{-1} R \begin{bmatrix} E^i_x \\ E^i_y \end{bmatrix}
$$

$$
= (C/\sqrt{2}) \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{23}
$$

$$
= (C/\sqrt{2}) \begin{bmatrix} 1 \\ 1 \end{bmatrix} .
$$

Again the scattered field is linearly polarized, with equal left and right components. The reflected field will always be polarized with its electric vector parallel to the wire (that is, in the x-direction) regardless of the state of polarization of the incident field. Obviously, if the incident field is polarized in the y-direction, there will be no reflected field.

As we have already mentioned, the scattering matrix is time independent, yet there is a temporal variation in the position of the wire. To take account of this temporal variation, we apply Eq. (9). In the case of the rotating wire, the receiver and wire were fixed so that the term exp (-ikz) was a constant which we could disregard. With the vibrating wire this term is a function of time and so must be considered. Equation (9) becomes

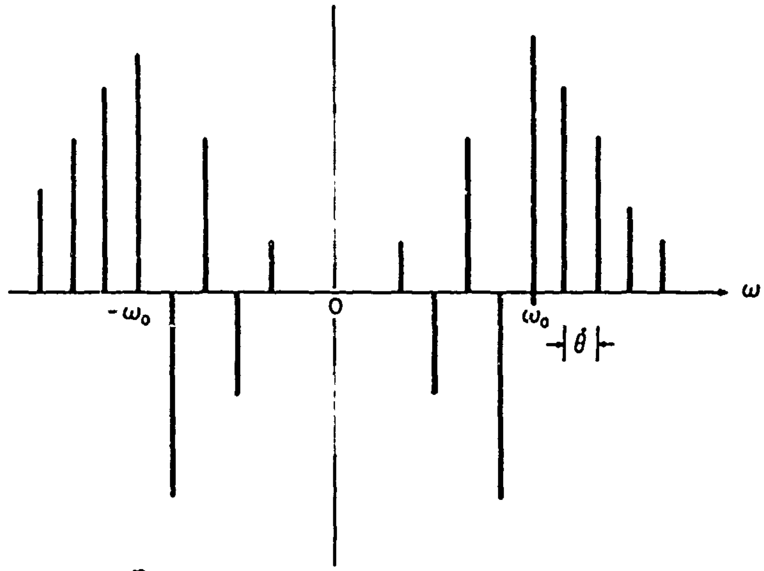
$$
\underset{\sim}{E}^s = \left[E^s_L \hat{L} + E^s_R \hat{R} \right] \exp \left\{ -i \left[k z_o \sin(\dot{\theta} t) - \omega_o t \right] \right\} . \tag{24}
$$

Equation (24) shows clearly that the effect of vibration on the scattered field is a phase modulation, whereas that of rotation is an amplitude modulation.

The real part of the exponential of Eq. (24) is a term proportional to $\cos\left[k z_o \sin(\dot{\theta} t) - \omega_o t \right]$, the Fourier transform of which gives the spectral dependence of the scattered field. Since the reflection matrix is time independent, it can be disregarded as an influence on the spectrum. The Fourier transform of the real part of the exponential is plotted in Figure 5 (see Champeney[3]),

3. Champeny, D.C. (1973) Fourier Transforms and Their Physical Application, Sec. 2.5, p.36. Academic Press, New York.

$$f(t) = A\cos[\, kz_0 \sin(\dot{\theta} t) + \omega_0\, t]$$



$$F(\omega) = \pi A \sum_{n=-\infty}^{\infty} \left\{ J_n(kz_0)\, \delta(\omega - \omega_0 - n\dot{\theta}) + J_n(kz_0)\, \delta(\omega + \omega_0 + n\dot{\theta}) \right\}$$

Figure 5. Spectral Dependence of a Phase Modulated Field. Each line is a Bessel function of order n, $J_n(kz_0)$

$$f(t) = \cos\left[kz_0 \sin(\dot{\theta} t) - \omega_0 t\right], \text{ and}$$

$$F(\omega) = \pi \sum_{n=-\infty}^{\infty} \left\{ J_n(kz_0)\, \delta(\omega - \omega_0 - n\dot{\theta}) + J_n(kz_0)\, \delta(\omega + \omega_0 + n\dot{\theta}) \right\}. \tag{25}$$

The spectrum is an infinite series of equispaced lines, each line being proportional to a Bessel function of order n where n runs from minus to plus infinity. The n^{th} side line measured from ω_0 is proportional to $J_n(kz_0)$.

An alternative derivation shows a clear physical relation between the modulation and the Doppler effect. From Eq. (19) the velocity of the wire is

$$v = \dot{\theta} z_0 \cos(\dot{\theta} t), \tag{26}$$

which gives a Doppler shift

$$\Delta\omega = \omega_0 v/c = (\omega_0 \dot\theta z_0/c) \cos(\dot\theta t). \tag{27}$$

Since frequency may be interpreted as a time rate of change of phase, we write

$$\dot\phi = \omega_0 \pm \Delta\omega = \omega_0 \pm (\omega_0 \dot\theta z_0/c) \cos(\dot\theta t). \tag{28a}$$

When integrated, Eq. (28a) gives the phase as

$$\phi = \omega_0 t \pm (\omega_0 z_0/c) \sin(\dot\theta t). \tag{28b}$$

The spectral dependence is obtained by taking the Fourier transform of cos ϕ as before. In Eq. (28b) ω_0/c equals k_0, so that the results of the two approaches are identical. It is also worth noting (and not unexpected) that the phase ϕ is directly proportional to the displacement amplitude z_0. Therefore the greater z_0, the greater the effect—all other factors being equal.

For a further discussion of the relation between the Doppler effect and rotating or vibrating wires, see Appendix B.

4. DISCUSSION

The body of this report has been concerned with the backscattered fields and their spectra, obtained from short wires performing periodic motions. By choosing the "short" wire, we simplified the problem and reduced the scattering to that from a point dipole or simple Rayleigh scatterer. The motions examined are rotation and vibration. A wire undergoing these motions provides a scattering model applicable to the complicated radar returns which comprise the identification problem.

Given the short wire as the scatterer, we find that the spectrum from the rotating wire is distinct from that of the vibrating wire. In addition, the spectrum from the rotating wire depends on the state of polarization of the incident field, unlike that of the vibrating wire for the geometrics considered here.

Rotation produces an amplitude modulation. For a linearly polarized incident field there are two side lines separated from the original frequency by plus or minus twice the rotation frequency. For a circularly polarized incident field only one side line appears, also displaced by twice the rotation frequency.

In contrast, vibration produces a phase modulation. The spectrum from the vibrating wire contains an infinite series of equispaced lines about the incident frequency line. The interline spacing is the vibration frequency, and the strength of the n^{th} line is proportional to the n^{th} order Bessel function $J_n(kz_o)$, where z_o is the vibration amplitude. Both spectra, therefore, provide the period of the relevant motion.

There are, of course, several possible approaches one might take to study the identification problem. The one we have taken here is quite simple, yet has a physical interpretation that is easy to grasp. (A large part of this interpretation is contained in Appendices B, C, and D.) From the characteristic spectra we can begin the study as a problem in signatures.

Appendix A

The Polarization Scattering Matrix and the Decomposition of the Electric Field

When a target scatters an incident wave, the linearity of Maxwell's equations imposes a linear relationship between the incident and scattered waves. A succinct and lucid description of this relation is achieved through the scattering matrix which is written generally as

$$\begin{bmatrix} E^s_{x'} \\ E^s_{y'} \end{bmatrix} = \frac{\exp\left[i\,\underset{\sim}{k}\cdot\underset{\sim}{R}\right]}{\sqrt{4-R^2}} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} E^i_x \\ E^i_y \end{bmatrix} . \tag{A1}$$

In Eq. (A1) $\underset{\sim}{k}$ is the wave vector and R is the distance from the target to the point at which the scattered field is observed. The matrix [A] with elements a_{ij} is the scattering matrix, and the incident and scattered fields are described as column matrices defined in terms of orthogonal x and y components. Kennaugh[4] describes the matrix and its applications at length.

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4. Kennaugh, E.M. (1966) Antenna and Scattering Theory: Recent Advances 1:1-17, The Ohio State University, Columbus, Ohio.

If we have a monostatic system, so that receiving and transmitting antennas are identical, we have the case of backscattering and the orthogonal bases of the two fields are the same. Equation (A1) becomes

$$\begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} = C \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} \tag{A2}$$

in which the proportionality factor is written as C; the matrix is now called "R" (for reflection) with matrix elements r_{ij}, and x and x' are identical directions as well as y and y'.

Physically, the scattering process changes the polarization characteristics of the radiation. For example, if the incident radiation is linearly polarized, the scattered radiation may have any state of polarization depending on the peculiar properties of the target. These peculiar properties include, of course, the state of motion of the target. The motion of the target leading to changes in what is seen by the incident radiation means that the matrix "R" now varies with time.

Before continuing with the determination of "R", we first formulate the description of the fields in terms of orthogonal polarization states. Our treatment of the polarization follows that of Jackson.[5] Since the propagation of light is rectilinear and is a transverse wave phenomenon, the electric vector is always located in a plane normal to the propagation vector. Therefore, the electric vector may be described in terms of two linearly independent unit vectors. We shall use $\hat{x}$ and $\hat{y}$ to denote unit polarization vectors in the x and y directions. As we shall see, we can also construct a pair of complex unit orthogonal vectors $\hat{L}$ and $\hat{R}$ that correspond to left and right circular polarization respectively and are related to $\hat{x}$ and $\hat{y}$ through a straightforward transformation.

The electric field $\underset{\sim}{E}$ of a wave propagating in the z-direction may be written as

$$\underset{\sim}{E}(z,t) = \left[E_x \hat{x} + E_y \hat{y}\right] \exp\left[i(kz - \omega_0 t)\right], \tag{A3}$$

where $\underset{\sim}{k}$ is the wave vector and ω_0 is the circular frequency. The amplitudes E_x and E_y are complex quantities which allow for a possible phase difference between the x and y components of the field. If E_x and E_y have the same phase, the wave is linearly polarized with the resultant electric vector $\underset{\sim}{E}$ oriented at an

5. Jackson, J.D. (1962) Classical Electrodynamics, Chap. 7, p. 202. John Wiley and Sons Inc., New York.

angle θ with the x-axis such that

$$\theta = \tan^{-1}(E_x/E_y). \tag{A4}$$

If a phase difference exists between E_x and E_y, the wave is elliptically polarized.

A particular case is that for which E_x and E_y are equal in magnitude, but their phases differ by $\pi/2$ rad. Equation (A3) becomes

$$\underset{\sim}{E}(z, t) = E_o(\hat{x} \pm i\hat{y}) \exp\left[i(kz - \omega_o t)\right]. \tag{A5}$$

Taking the real part of Eq. (A4), we find for the components of the field

$$E_x = E_o \cos(kz - \omega_o t), \text{ and}$$

$$E_y = \mp E_o \sin(kz - \omega_o t). \tag{A6}$$

Facing the wave, that is looking along the negative z-direction towards the propagating wave, what does one see? Taking the positive sign in Eq. (A5), (and therefore the negative in Eq. A6), we find that the electric vector has a constant magnitude and rotates with frequency ω_o in the counterclockwise sense. This polarization state is designated as left circularly polarized (LCP). Taking the negative sign in Eq. (A5), we find constant magnitude for the electric vector, as before, but a clockwise rotation. This we call right circularly polarized (RCP).

To describe a general state of polarization, we have used two orthogonal unit vectors, $\hat{x}$ and $\hat{y}$, as our basis. An equally valid basis is the circular pair, $\hat{L}$ and $\hat{R}$ defined as

$$\hat{L} = (1/\sqrt{2})(\hat{x} + i\hat{y}), \text{ and}$$

$$\hat{R} = (1/\sqrt{2})(\hat{x} - i\hat{y}) \tag{A7a}$$

with properties

$$\hat{L}^* \cdot \hat{R} = \hat{R}^* \cdot \hat{L} = 0 \text{ and}$$

$$\hat{L}^* \cdot \hat{L} = \hat{R}^* \cdot \hat{R} = 1. \tag{A7b}$$

The vector $\hat{L}$ corresponds to pure LCP and the vector $\hat{R}$ to pure RCP. Solving Eq. (7) for $\hat{x}$ and $\hat{y}$ leads to the inverse transformations

$$\hat{x} = (1/\sqrt{2})\,(\hat{L} + \hat{R}) \text{ and}$$

$$\hat{y} = (i/\sqrt{2})\,(-\hat{L} - \hat{R}). \tag{A8}$$

The circular basis is fully equivalent to the cartesian basis, $\hat{x}$ and $\hat{y}$, for the wave description. In terms of the circular basis Eq. (A3) becomes

$$\underset{\sim}{E} = \left[E_L \hat{L} + E_R \hat{R}\right] \exp\left[i(kz - \omega_o t)\right], \tag{A9}$$

where E_L and E_R are complex amplitudes. In general, we can write the ratio of E_L and E_R as

$$E_R/E_L = r \exp(i\alpha). \tag{A10}$$

In Eq. (A10), r denotes the ratio of the magnitudes of the two amplitudes, and α is their phase difference.

If E_L and E_R differ in magnitude, (that is, r is not unity), but their phase difference is zero, Eq. (A10) describes an elliptically polarized wave. The principal axes of the ellipse described by the electric vector are in the directions of $\hat{x}$ and $\hat{y}$. The ratio of the semi-major to the semi-minor axis is $(1+r)/(1-r)$.

If the phase difference α between E_L and E_R is non-zero, the axes of the ellipse are rotated by $\alpha/2$.

If the phase difference is zero, and the two amplitudes are equal (r equals unity), the wave is linearly polarized.

Lastly, if either E_L or E_R is zero, the wave is circularly polarized.

The matrix formulation of these transformations greatly facilitates our calculations. We start with the equivalence of the two bases $(\hat{x}, \hat{y})$ and $(\hat{L}, \hat{R})$ and define the two transformation matrices, $[Q]$ and its inverse $[Q^{-1}]$, as

$$Q = (1/\sqrt{2})\begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix},$$

$$Q^{-1} = (1/\sqrt{2})\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \tag{A11a}$$

such that

$$QQ^{-1} = I \tag{A11b}$$

where I is the identity matrix. Writing the amplitudes as column matrices, we can apply the matrices of Eq. (A11a) to write the transformations of any wave as

$$\begin{bmatrix} E_L \\ E_R \end{bmatrix} = Q^{-1} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

and (A12)

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = Q \begin{bmatrix} E_L \\ E_R \end{bmatrix}.$$

With Eq. (A12) we can transform easily from the cartesian to the circular amplitudes, and vice versa for any arbitrary wave.

In Eq. (A2) we have related the cartesian amplitudes of the scattered and incident waves through the scattering matrix [R], where [R] is the cartesian scattering matrix. Using Q and Q^{-1}, we can compute the circular scattering matrix that relates the circular amplitudes of the scattered and incident waves. If the incident field is given in terms of the circular basis, its cartesian components are

$$\begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = Q \begin{bmatrix} E_L^i \\ E_R^i \end{bmatrix} . \tag{A13}$$

The scattered field is then

$$\begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix} = C R \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix} = C R Q \begin{bmatrix} E_L^i \\ E_R^i \end{bmatrix} \tag{A14a}$$

with C a proportionality factor. Since

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = Q^{-1} \begin{bmatrix} E_x^s \\ E_y^s \end{bmatrix}, \tag{A14b}$$

we find as our final equation for the circular components of the scattered field

$$\begin{bmatrix} E_L^s \\ E_R^s \end{bmatrix} = C\, Q^{-1} R\, Q \begin{bmatrix} E_L^i \\ E_R^i \end{bmatrix} \tag{A15}$$

Therefore, the circular scattering matrix is $Q^{-1} R Q$.

Equation (A15) applies to any scattering matrix [R]. We have imposed no restrictions on it, but have provided a way of going from the cartesian to the circular form, once [R] is known.

Appendix B

The Doppler Effect and Scatterers With Periodic Motion

In the Doppler effect, as evidenced by the use of radar reflections to determine target velocities, the motion of the target introduces a change of frequency in the scattered radiation. This frequency change is a function of the relative velocity between target and radiation source. Measurement of the change provides information about the target velocity. In this report, we have discussed frequency changes occurring with rotating and moving wires. The first is an amplitude modulation, the second a phase modulation. Are we justified in subsuming these frequency changes caused by periodic motions of a target under the general class of Doppler phenomena?

B.1 THE ROTATING WIRE AND THE DOPPLER EFFECT

The normal Doppler effect is radial, that is, the scalar product between the target velocity and the radar propagation vector is non-zero. If the product is zero, there is no effect. This is equivalent to the statement that there is no classicial transverse Doppler effect. This is illustrated by Sommerfeld[6]

6. Sommerfeld, A. (1954) Optics, Chap. 11, Sec. 13, p. 72, Academic Press, New York.

in his discussion of reflection from a moving mirror. If the mirror has a velocity component along the propagation direction, the frequency changes upon reflection. If, however, the mirror moves at right angles to the propagation direction, the light is reflected with no change in frequency. To explain this, one must remember that the wire (unlike the mirror) is not an invariant when rotating. A large plane mirror with a uniform reflecting surface presents a constant aspect to the incident radiation, when moving transversely. The rotating wire is, however, continually changing its aspect. Were the mirror rotating, there would be no frequency change. We conclude that reflection by a short wire, rotating in a plane normal to the propagation direction, does have an effect which may be regarded as a classical transverse Doppler shift.

It is important to stress that this shift is not related at all to the relativistic transverse Doppler effect,[7] which is a consequence of the fact that clocks run differently when viewed from different inertial systems. The relativistic transverse Doppler effect is second order in v/c, where v is the relative velocity between object and source and c is the velocity of light. The effects described in this report, however, are first order in v/c.

B.2 THE VIBRATING WIRE AND THE DOPPLER EFFECT

The frequency change caused by periodic vibration is a normal Doppler phenomenon. In Section 3, the phase modulation is derived within the Doppler framework (cf. Eqs (26) - (28)). The direction of vibration assumed in Section 3 is always parallel (or anti-parallel) to the propagation direction. If the wire vibrated in the xy-plane, normal to the propagation direction, there would be no effect on the frequency of the reflected radiation. The only effect would be on the position of the return in the xy-plane. In this sense, the frequency change associated with vibration is simpler than that due to rotation, since it can be interpreted as a straightforward periodic Doppler effect.

7. Møller, C. (1972) The Theory of Relativity, 2nd ed. Sec. 2.11, p. 59, Clarendon, Oxford, England.

Appendix C

The Relation Between Raman Scattering and the Spectra of the Backscattered Field From Wires With Periodic Motion

The spectrum of the field scattered from the rotating wire is characterized in general by three lines, one at the frequency ν_o of the incident field and two displaced symmetrically by twice the rotation angular frequency from ν_o. This spectrum is strongly reminiscent of that of Raman scattering from a rotating molecule. Moreover the spectrum from the vibrating wire has similarities to that from an oscillator which is the simplest model of a vibrational Raman scatterer. Close examination shows clearly that these similarities are not fortuitous. However, there are definite differences between scattering from macroscopic wires and molecules which preclude a one-to-one correspondence. A study of both similarities and differences is helpful–indeed, very helpful–in understanding the physical meaning of the two phenomena.

C.1 THE RAMAN EFFECT AND THE ROTATING WIRE

Raman scattering from rotating molecules is a quantum mechanical phenomenon, although a classical theory exists which does offer considerable

insight. Detailed discussions of the effect are found in both Herzberg[8] and Kohlrausch.[9] The classical theory is given by Cabannes and Rocard.[10]

The quantum explanation of the effect is described simply. An incident photon of energy $\hbar \nu_o$ interacts with a rotating molecule. It can be scattered so that its frequency and, therefore, its energy remain unchanged. However, scattering with a change in frequency is also possible. The scattering molecule has discrete rotational energy levels. If the molecule absorbs energy from the incident photon (so that the molecule is raised to a higher energy level), the scattered photon has less energy and, therefore, a smaller frequency than does the incident one. The converse is also true. The interaction may be such that the molecule loses energy and drops to a lower level. This energy is taken up by the radiation field so that the scattered photon has greater energy and, therefore, a higher frequency than those of the incident photon. The frequency ν arising from absorption (or emission) is related to the energy difference between the two levels as

$$\Delta E = \hbar \nu . \tag{C1}$$

These levels are, of course, discrete and their eigenvalues are determined by solution of the relevant Schrödinger equation which contains the rotational kinetic energy term. For the rigid rotator these levels are

$$E = \hbar^2 J(J+1)/2I, \tag{C2}$$

where J is the rotational quantum number and has the integral values, 0, 1, 2, ... and I is the moment of inertia. The selection rules for the Raman transitions are

$$\Delta J = 0, \pm 2, \tag{C3}$$

and the intensity of the transition depends on the change of the molecular polarizability in a fixed direction during the rotation. When J equals zero, there is no energy change and the undisplaced line is observed.

The classical theory presupposes a change in the polarizability arising from the molecular rotation. The rotation affects the polarizability so that

$$\alpha = \alpha_o + \alpha_1 \exp(i 2\pi \nu_R t), \tag{C4}$$

8. Herzberg, G. (1950) Molecular Spectra and Molecular Structure: I. Spectra of Diatomic Molecules, Chap. III, p. 66, Van Nostrand, New York.

9. Kohlrausch, K. W. F. (1931) Der Smekal-Raman Effekt, Springer, Berlin, Germany.

10. Cabannes, J., and Rocard, Y. (1929) J. Phys. Rad. 10:52, Paris, France.

where α_0 is the average polarizability, α_1 is the amplitude of the periodic change caused by rotation and ω_R is the angular frequency of rotation. The factor 2 appears because the polarizability is the same for the molecule rotated through 180° as before. An incident field $\underset{\sim}{E} \exp(i\omega_0 t)$ induces a time-dependent dipole moment P,

$$\underset{\sim}{P} = \alpha \underset{\sim}{E} \exp(i\omega_0 t), \tag{C5}$$

which contains terms proportional to $\exp\left[i(\omega_0 \pm 2\omega_R)t\right]$ as well as $\exp(i\omega_0 t)$. Thus both the classical and quantum theories multiply the rotation frequency by 2 so that $2\omega_R$ appears in the scattered spectrum as it does for the rotating wire. (The 2 in the quantum theory arises from the selection rule, Eq. (C3).)

In the classical theory, there are no restrictions on the values ω_R can have. Unlike the quantum theory, all frequencies are allowed. The classical theory is obviously very close to that used in Section 2 for the rotating wire. We may, therefore, regard the rotating wire as equivalent to a rotating macromolecule. There are at least two significant differences, however, between this macromolecule and a rotating molecule obeying quantum theory. The rotating wire considered as a molecule does not have a set of discrete energy levels. Its kinetic energy T is related to the velocity of rotation ω_R as

$$T = 1/2\, I\, \omega_R^2, \tag{C6}$$

which is quite different from Eq. (C2). Since T is not quantized, neither is ω_R. The energy is a continuum.

More important, perhaps, is the fact that the wire cannot be considered apart from its source of power. This also differs from the molecular case for which we regard the molecule (at least to the first approximation) as being isolated. The power source or motor drives the wire at constant angular velocity. If energy is supplied to the wire by the incident wave (a decrease in scattered frequency), the motor does a little less work. If energy is supplied by the wire to the scattered field (an increase in scattered frequency), the deficiency is made up by the motor. In effect, the energy of the wire never changes or, equivalently, the macromolecule remains in the same energy level, independent of the scattering process. This contrasts with the molecular Raman effect which leaves the molecule in a different energy state.

A third point which should be remembered is that the orientation of the rotation axis of the wire to the propagation direction of the field is at our disposal,

whereas orientation of molecules is difficult and not possible for all cases. Hence Raman scattering from molecules is often from a randomly oriented assembly, while scattering from a rotating wire provides a simpler well-defined system, especially as regards polarization effects.

C. THE RAMAN EFFECT AND THE VIBRATING WIRE

Raman scattering from vibrating molecules is also a quantum mechanical phenomenon. The simplest vibrating molecule is diatomic; each atom moves with respect to the other in simple harmonic motion. Such motion is equivalent to harmonic motion of the reduced mass about an equilibrium position. This is represented by a harmonic oscillator whose Schrödinger equation is readily solved. The energy eigenvalues of the solution are

$$E(n) = \hbar\omega_{osc}(n + 1/2), \tag{C7}$$

where ω_{osc} is the oscillation frequency. Transitions between these levels lead to energy changes in the oscillator and, hence, in the radiation field. The selection rules for allowed Raman transitions are $\Delta n = \pm 1$; that is, transitions are allowed only between adjacent vibrational states. Therefore, illumination with light of frequency ω_o will produce a spectrum containing a line at ω_o and two Raman lines at $(\omega_o \pm \omega_{osc})$.

The classical theory of Cabannes and Rocard[10] interprets the effect in terms of a time-varying molecular polarizability

$$\alpha = \alpha_o + \alpha_1 \exp(i\omega_{osc} t), \tag{C8}$$

where α_o is the average polarizability, as before, and α_1 is the amplitude of the polarizability change caused by vibration. The resultant dipole moment induced by the incident field will contain terms proportional to $\exp\left[i(\omega_o \pm \omega_{osc})\right]$ and $\exp(i\omega_o t)$. These three frequencies will appear in the spectrum of the scattered light. This differs from the vibrating wire which produces a scattered spectrum with an infinite series of lines separated by frequency increments ω_{osc}.

The vibrating wire examined in this report is less closely related to the vibrating molecule than is the rotating wire to the rotating molecule. The molecular vibrations correspond to internal vibrations within the molecule–unlike the wire, the molecule is not rigid. These internal deformations are responsible for the change in polarizability. The rotating molecule may, however, be looked at as

essentially rigid. Therefore, for rotation the analogy between the wire and molecule is considerably closer than for vibration.

Appendix D

Energetic and Momentum Considerations

The interaction between the incident field and the scattering wire can lead to an exchange of energy and of angular momentum between the wire and the field. We know from the de Broglie relation that energy is a function of frequency.

$$E = \hbar\omega . \qquad \text{(D1)}$$

The symbol $\hbar$ is Planck's action constant divided by 2π. Therefore, if the frequency of the scattered light differs from that of the incident by $\Delta\omega$, its energy must also differ, and by the amount $\hbar\Delta\omega$.

A second point made by quantum theory is that light particles always carry angular momentum. Photons may exist in either one of two states, LCP or RCP. An LCP photon has a positive helicity and an angular momentum $+\hbar$. Similarly, an RCP photon has a negative helicity and an angular momentum $-\hbar$. These are the only two allowed values of photon spin or intrinsic angular momentum. Moreover, spin angular momentum is frequency independent. No matter what the photon frequency may be (from radio waves to γ-rays), a single photon can have angular momentum $\pm\hbar$ only. Nonetheless, the net angular momentum of linearly polarized light is zero. This is because linearly polarized light consists of equal

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numbers of LCP and RCP photons. If a beam is linearly polarized with N photons of each type, the total angular momentum is thus $+N\hbar$ plus $-N\hbar$ or zero.

In all the cases we have considered, our scatterer has been a short wire. The reflection from such a wire (whether rotating, vibrating, or stationary) is always linearly polarized, regardless of the state of motion of the wire. (For rotation $\dot{\theta}$ must be small compared with ν_0, a condition which holds for all realizable mechanical rotation rates.) This linear polarization is a consequence of scattering from the linear short wire, which acts as a point dipole. However, one must be careful in discussing the backscattered wave. It is true that the reflected radiation is always linearly polarized, and in the direction parallel to the orientation of the wire at the instant of reflection. For the rotating wire, this orientation is not constant with time. The direction of polarization rotates with twice the velocity of the wire. The doubling of the velocity is a consequence of the fact that in half a period the wire has returned to its original state as seen by the incident radiation. For example, if the reflected wave is linearly polarized in the x-direction at a given time, the wave reflected a quarter of a period later $(\pi/2\dot{\theta})$ is linearly polarized in the y-direction—provided, of course, that the electric vector of the incident wave has a y-component. At a time half a period later $(\pi/\dot{\theta})$, the reflected wave is again polarized in the x-direction.

If the incident field is circularly polarized, it carries net angular momentum. Yet those photons which are backscattered form a linearly polarized field. If N photons (all LCP) are scattered, $N\hbar$ units of angular momentum have disappeared from the radiation field and must have been taken up by the wire. From this we conclude that the radiation field has exerted a torque on the wire, during the scattering. Therefore, work has been done on or by the mechanical system of the wire and its driving motor.

Let us consider the Examples of Sections 2 and 3.

Rotation-Example 1: The incident field is LCP and is monochromatic with frequency ν_0. The backscattered field is linearly polarized and has lines at ν_0 and $(\nu_0 + 2\dot{\theta})$. Therefore, the incident field has exerted a torque on the wire which has resulted in a loss of field angular momentum. Secondly, the wire (and therefore its driving motor) has supplied energy to the scattered field, thereby increasing the frequency of part of the field from ν_0 to $(\nu_0 + 2\dot{\theta})$.

Rotation-Example 2: Both the incident and scattered fields are linearly polarized so that there has been no exchange of angular momentum. Moreover, there are an equal number of photons having the two frequencies $(\nu_0 + 2\dot{\theta})$ and $(\nu_0 - 2\dot{\theta})$. If there are N with each frequency, their total energy is $2N(\hbar\nu_0)$. But this is precisely the energy of the incident photons which were scattered. Therefore, there has been no exchange of energy between field and wire.

D.1 VIBRATING WIRE

The angular momentum considerations depend on the state of polarization of the incident field. The reflected field is always linearly polarized, so that it carries no net angular momentum. If the incident field is also linearly polarized, the net angular momentum of the radiation field before and after scattering is zero. If the field is circularly polarized, it has net angular momentum which is lost in the scattering process. This is evidenced as a torque acting upon the vibrating wire which must be balanced by a counter-torque exerted by the driving mechanism to prevent rotation of the wire. This represents work done by the driving mechanism, in other words, an energy current flowing to or from the wire.

The spectrum of Figure 4 is symmetric about the frequency of the incident field ω_o apart from the signs of some lines. The intensity of a given line is proportional to the square of that spectral field line; therefore, the energy spectrum is truly symmetrical about ω_o. If we sum over all the lines, therefore, we find that the total scattered energy does not differ from that of the incident field which has been scattered, even though it is distributed over an infinite number of lines, all but one of which has a frequency different from the incident frequency. Note that any energy changes in the radiation field must involve the motor drive of the wire, either as an energy sink or source. This differs from the molecular Raman scattering in which the energy change involves a quantum jump of the molecule from one level to another as we discussed in Appendix C.

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